

## **A comparison of heavy-tailed VaR estimates and Filtered Historical Simulation: Evidence from emerging markets**

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*The purpose of this paper is to investigate relative performance of Value at Risk (VaR) models in forecasting risk of emerging markets financial series. Among well-known Value-at-Risk modelling techniques, we based our estimates on GARCH, Extreme Value Theory (EVT) and Filtered Historical Simulation (FHS). Risk estimates are then exposed to the backtesting models in order to evaluating comparative accuracy of the VaR models. The results indicate that EVT-based VaR estimates provides us with more accurate estimation of VaR specially in higher quantiles.*

Field of Research: Finance, Risk Analysis

### **1. Introduction**

It is not a long time since the world financial system is recovering from its latest crisis known as the USA subprime crisis that we are again dealing with a new one occurring in the Euro zone. The severity of the aforementioned phenomena has stressed the importance of understanding the assumptions and weaknesses of the different risk management methodologies, hence managers who had not considered such assumptions, faced with greater losses than the ones estimated by their quantitative systems (German, 1998). Weak supervision and management of financial risks is assumed to be the main cause of these financial crises. In response to financial failures the Value-at-Risk (VaR) was introduced in the 1990s to play more imperative role for risk measuring (Cheong, 2008). Value at risk represents all of the risks in a portfolio with a single number used in reporting and also it is straightforward to understand. The VaR with a given level of confidence summarizes the worst loss over a specific horizon (Gençay and Selçuk, 2004).

In a VaR context, precise prediction of the probability of an extreme movement in the value of a portfolio is essential for both risk management and regulatory purposes. By their very nature, extreme movements are related to the tails of the distribution of the underlying data generating process. Several tail studies, after the pioneering work by Mandelbrot, indicate that most financial time series are fat-tailed. (Gençay, Selçuk and Ulugülyağci, 2003). To resolve fat-tailed distribution problem, statisticians have elaborated a tailor-made approach-extreme value (EV) which found its identity from an underlying theory-Extreme Value Theory. The key to this approach is a theorem—the extreme value theorem—that tells us what the limiting distribution of extreme values should look like (Dowd, 2006). So the approach

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equipped with sufficient instrument to focus on tail estimation which is desirable in VaR literature.

In addition to fat-tailness, we know that behind stochastic volatility in financial time series, there exists strong empirical evidence which implies that volatilities are not necessarily independent over time. This Phenomena is often explained by concepts such as heteroskedasticity, volatility cluster and volatility persistence which have significant role in risk modeling. The literature commonly describes persistence in time series volatility using ARCH or GARCH models that give rise to unconditional symmetric and leptokurtic distributions. Here leptokurtosis follows from persistence in the conditional variance, which produces the clusters of low volatility and high volatility returns (Billio and Pelizzon, 2000).

So we can say, VaR literature contains a lot of models born to consider these important features of financial series. Each of models is based on an approach by which the main assumptions of risk modeling are determined. The approaches are often categorized into three major groups including parametric, nonparametric and semi-parametric (Radpour and Abdoh, 2010). Nonparametric approach relaxes all limiting distributional assumptions in parametric one and It is often better to enrich non-parametric models with another corresponding approach which may have the flexibility of conditional volatility models (Barone-Adesi and Giannopoulos,2001). These issues in literature introduce semi-parametric methods. Hull and White and Barone-Adesi et al. (Barone-Adesi and Giannopoulos,1999) combined different methods to put forward Filtered Historical Simulation (FHS) in this field.

There are plenty of numbers of estimating VaR models available and comparing the performance of these models has always been the area under discussion. (Gençay and Selçuk, 2004, Aktham, 2006, Giannopoulos and Tunaru, 2005) While few empirical works have conducted a comparison between parametric and semi-parametric models so far (Žiković and Aktan, 2009, Ghorbel and Trabelsi, 2007). In our study we compared three different models for estimating VaR. Two of these models are based on parametric approaches with heavy-tailed consideration (GARCH and EVT) and one of them is a semi-parametric approach (FHS). We fitted a Student's t-distribution on data, conditional on a GARCH volatility process to capture tail heaviness and volatility clustering. The great advantage of the t-distribution over the normal is its ability to handle reasonable amounts of excess kurtosis. The combination of the fitting of GARCH model to estimate the volatility with extreme value theory (EVT) for estimating the tail of the innovation distribution of the model is another parametric approach which was used in our study to estimate conditional quantiles. Filtered Historical Simulation (FHS) is the promising semi-parametric model with combination of the benefits of Historical Simulation and conditional volatility models (Dowd,2006) which is considered as one of the best tools for estimating VaR by Zenti and Pallotta (Zenti and Pallotta,2001).

In this paper, we focused on main indices of emerging markets. Most of countries with an emerging market have large resources and population and they have prominent effect on their neighbor countries. Emerging markets have a small share in the world economy and volatility is the inherent part of them. The structure of emerging markets is different from developed countries and dynamics of market, low liquidity and financial shocks are serious challenges in these markets. We selected BUX index of Hungary, PX index of Czech Republic and RTS index of Russia for

European emerging markets. NSEI index from India, TWII weighted index of Taiwan, JKSE index of Indonesia And finally SSE composite index of China are among Asian emerging markets which we considered for testing our models. The upper bound of the related time series of all indices limit to the beginning of 2010 but they don't have the same lower bounds. Data were gathered from each stock market's database.

Our test results indicate that the number of violations occurred under conditional EVT method is closer to the expected number of violations comparing to FHS and conditional t-distribution estimations. Either FHS or conditional t model cannot perform as well as EVT model in estimating the VaR in different levels of confidence.

The rest of this paper is organized as follows: in [Section 2](#) we present methodology framework for our study; presenting our preliminary data analysis and empirical evidence in [Section 3](#) we finally put forward our conclusion in [Section 4](#).

## 2. Methodology

Assume that  $x_t$  represents the actual daily return of the stock index and we aim to estimate the targeted quantiles in predictions of return distributions for future days. In fact we are interested in this measure,

$$x_q^t = \mu_{t+1} + \sigma_{t+1}z_q \tag{1}$$

Where  $\sigma_t$  and  $\mu_t$  are the volatility and the expected return of the  $x_t$  on day  $t$  respectively, and  $z_q$  is the lower  $q$ th quantile of return distribution. In order to reach estimation for the measure, we should choose a particular model for the dynamics of mean and volatility.

In this paper we use a first-order autoregressive model AR (1) for estimating  $\mu_{t+1}$ . In AR (1) model, which is used in predictions of financial time series, the process of developing the measure is

$$x_t = \alpha_0 + \alpha_1x_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t \sim n(0, \sigma^2) \tag{2}$$

Where  $\alpha_0$  and  $\alpha_i$  are the parameters of the model and can be estimated through maximum likelihood estimation (MLE) (for more info on MLE see (Sarabia and Prieto, 2009)), and  $\varepsilon_t$  has a distribution function  $F$  with zero mean and variance  $\sigma_t^2$ . Therefore, applying AR (1) model, the conditional mean is given by

$$\mu_t = \alpha_0 + \alpha_1x_{t-1} \tag{3}$$

Many different models for volatility dynamics have been proposed in econometric literature including models from the ARCH/GARCH family (Bollerslev, 1992), HARCH processes (Muller, 1997) and stochastic volatility models (Shephard, 1996). Here we use GARCH (1, 1) for estimating  $\sigma_{t+1}$ . Hence, the conditional variance is given by

$$\sigma_t^2 = \omega_0 + \omega_1\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4}$$

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Where  $\omega_0, \omega_1$  and  $\beta > 0$ , and  $\omega_1 + \beta < 1$ . These parameters can also be derived from maximum likelihood estimation.

In order to pose a better modeling of fat-tailed distribution, which has some excess kurtosis from normal distribution, we fitted student's t-distribution on daily return data and apply GARCH-t model for tracking volatility process (Chien-Liang, 1996).

We can define  $z_q$  for t-distribution as follows

$$z_q = \sqrt{\frac{(v-2)}{v}} F_T^{-1}(q) \quad (5)$$

Where  $v$  is the degree of freedom for t-distribution and  $F_T(t)$  is the density function for this distribution.

In this model, VaR for the day  $t$  is given by

$$VaR_t = -P_{t-1}(\mu_t - \sigma_t z_q) \quad (6)$$

And  $P_{t-1}$  is the value of the sample data for the day  $t - 1$ .

We select the peak over threshold (POT) method within EVT to study tail behavior. Assume that we show the threshold by  $u$ , the probability distribution function of excess value of  $x$  over  $u$  is defined by

$$F_u(y) = \Pr\{x - u \leq y | x > u\} \quad (7)$$

Where  $y$  is the value of exceedances which is defined as  $y = x - u$ . We can rewrite the probability as

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} \quad (8)$$

As Balkema and de Haan (1974) and Pickands (1975) showed, when  $u$  becomes sufficiently large, the excess distribution  $F_u(y)$  converges to Generalized Pareto Distribution (GPD) which is defined as

$$G_{\xi, \sigma, \mu}(x) = \begin{cases} 1 - (1 + \xi \frac{x-\mu}{\sigma})^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{(x-\mu)}{\sigma}} & \text{if } \xi = 0 \end{cases} \quad (9)$$

With

$$x \in \begin{cases} [\mu, \infty] & \text{if } \xi \geq 0 \\ \left[ \mu, \frac{\mu - \sigma}{\xi} \right] & \text{if } \xi < 0 \end{cases}$$

Here,  $\sigma$  is the standard deviation and  $\xi$  is the tail index, which indicates the fatness of tail of data distribution. The bigger the  $\xi$  the heavier the tail is.

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If we isolate  $F(u + y)$  from the equation (8), we can write

$$F(u + y) = [1 - F(u)]F_u(y) + F(u) \quad (10)$$

Remember that  $x = y + u$  and  $u$  is large enough so  $F_u(y)$  can be approximated by GPD, then

$$F(x) = [1 - F(u)]G_{\xi, \sigma, \mu}(x - u) + F(u) \quad (11)$$

If we substitute current  $F(u)$  with  $\frac{n - N_u}{n}$ , in which  $n$  is the size of our sample and  $N_u$  is the number of data which are above threshold, we can calculate VaR for a given probability ( $p$ ) as

$$VaR = u + \frac{\sigma}{\xi} \left( \left( \frac{n}{N_u} p \right)^{-\xi} - 1 \right) \quad (12)$$

Filtered historical simulation (FHS) is a semi-parametric approach which tries to combine the advantageous of historical simulation method with those of conditional volatility models like GARCH. In implementing FHS, the first step is to fit a GARCH model to the sample data. Here we use AGARCH model, to estimate the volatility, which in addition to the characteristics of GARCH model, has a leverage effect (e.g. positive and negative return have different impact on volatility). An estimate of volatility can be obtained by

$$\sigma_t^2 = \omega_0 + \omega_1(\varepsilon_{t-1} + \gamma)^2 + \beta\sigma_{t-1}^2 \quad (13)$$

Where  $\gamma$  reflects an asymmetric effect on volatility whether the last period random error was positive or negative.

And  $r_t$ , the daily return, is calculated with first-order autoregressive AR (1) model, (equation (2)). In order to make residuals suitable for historical simulation, we should bring them closer to a stationary iid distribution. Hence, we have

$$z_t = \frac{\varepsilon_t}{\sigma_t} \quad (14)$$

Here, standardized residuals  $z_t$  are calculated through dividing residuals  $\varepsilon_t$  by conditional volatility forecast  $\sigma_t$  which is then bootstrapped to obtain a standardized historical time series.

That series of historical residuals are then updated with forecasted volatility  $\hat{\sigma}_{t+1}$  to obtain current market condition

$$\hat{z}_{t+1} = z_t \times \hat{\sigma}_{t+1} \quad (15)$$

We can generate simulated returns  $\hat{r}_{t+1}$

$$\hat{r}_{t+1} = \alpha_0 + \alpha_1 \hat{r}_t + \hat{z}_{t+1} \quad (16)$$

Therefore, we have some simulated data for tomorrow's return and considering VaR to be the loss corresponding to the chosen confidence level. We can calculate VaR for each simulated return series and the average on all, will give an estimation of desired VaR for the targeted day (Centeno and Wang, 2006, Barone-Adesi, 2000).

### 3. Findings and Data Analysis

In this study, we examined the specific VaR measuring models in seven different stock indices of emerging markets. We selected these markets according to FTSE Group classification in Europe and Asia (For further information see [www.ftse.com](http://www.ftse.com)).

Fig. 1 illustrates the corresponding daily price index and return for Russian trading system stock exchange in period of 1/9/1995 to 16/4/2010. Within 3000 to 3500 observations (years 2007 and 2008), high volatile price changes are obvious. There is no need to say that during this period Russia as well as other countries was faced with global financial crisis. The RTS index has fallen by around 50% from its peak in may 2008, turning back to the level of summer 2006. The similar patterns can be easily tracked in other indices in our study.

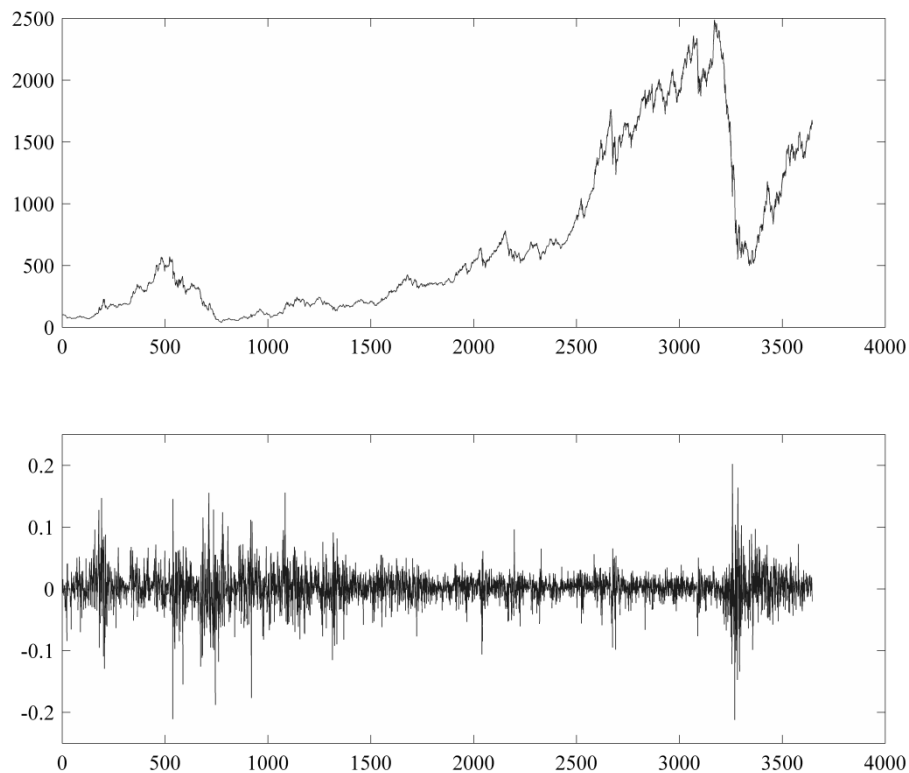


Fig. 1: Price index and daily return for RTS

In Table 1 some descriptive statistics for each stock index are shown. These statistics are based on the daily returns ( $r_t$ ) which are calculated from  $r_t = \log\left(\frac{x_t}{x_{t-1}}\right) \times 100$ , where  $x_t$  is the daily closing value of the stock market on day  $t$ .

The high average of daily return is a result of high inflation rate in corresponding country. In the observed period, the annual rate of inflation in Russia was relatively

high, comparing to other countries in this study, and the relatively high average return of this index can be explained by this fact.

The Jarque-Bera (JB) statistics as well as skewness and kurtosis values reject the null hypothesis that the daily returns of indices have normal distribution. The critical value for determining whether to reject the null hypothesis at the 5% level is 5.99 and it is obvious that JB test rejects the normal distribution hypothesis for all studied time series.

For better visual understanding in Fig. 2 we illustrated the quantile-quantile (Q-Q) plot for RTS daily return against the normal distribution and it can be viewed at a glance that the series show non-normal and heavy-tailed forms which are general between all considered stocks.

	Sample Size	Mean	Min	Max	Std	Kurtosis	Skewness	Jarque-Bera
<b>BUX</b>	3250	0.05	-18.0	13.6	0.019	13.23	-0.70	14477*
<b>PX</b>	4000	0.03	-16.2	15.4	0.016	17.51	0.35	35204*
<b>NSEI</b>	4500	0.06	-13.1	16.3	0.018	9.16	-0.09	7477*
<b>RTS</b>	3500	0.08	-21.2	20.2	0.029	9.73	-0.38	6968*
<b>TWII</b>	3000	0.00	-9.9	8.5	0.017	5.25	-0.14	672*
<b>SSE</b>	2500	0.03	-9.3	9.4	0.017	7.41	-0.09	2145*
<b>JKSE</b>	3000	0.04	-12.7	13.1	0.018	9.28	-0.14	5157*

Table 1: Descriptive statistics for daily return

\* Significance probability under 0.05.

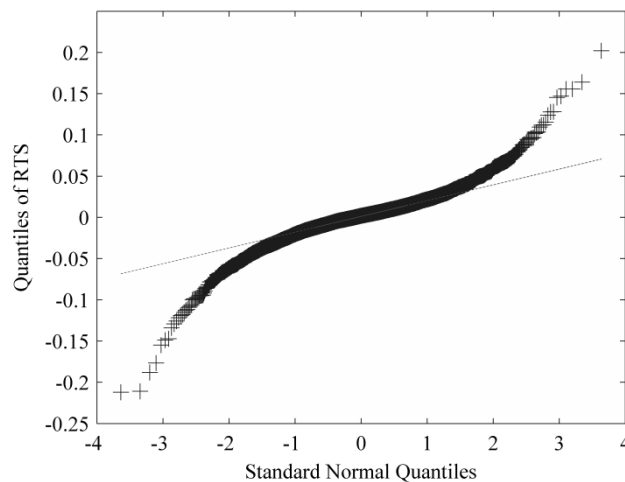


Fig. 2: Quantile-quantile plot of RTS empirical distribution versus the normal distribution

According to the statistics, the excess kurtosis apparently indicates that the daily return distributions have fat tails and negative values of skewness suggest that the left tails are remarkably extreme. This finding conducts us to implement models which consider heavy tails. The fitting of Student’s t-distribution on data is one method and using extreme value theory (EVT) for estimating the tail of the innovation

distribution (conditional on a GARCH volatility process) is another parametric method that both consider fat tails.

In Fig. 1 the clusters of volatilities can be detected. In Fig. 3 the correlograms for square return as well as raw data of daily return time series for RTS index are shown. With considering the critical values (dashed horizontal lines), it is obvious that the series are not independent and identically-distributed (iid). This result supports the implementation of autoregressive modeling in estimating value-at-risk which takes long-memory into account.

Moreover, the Ljung–Box test results for autocorrelation (Table 2) show that returns in our data set have considerable autocorrelation. Additionally, the Ljung–Box test statistics for autocorrelation in the square returns all indicate that the second-order moments are related. With comparison of critical values (43.77, 79.08, and 113.15 sequentially for 30, 60 and 90 lags), it can be concluded that square returns as well as raw daily returns are not iid.

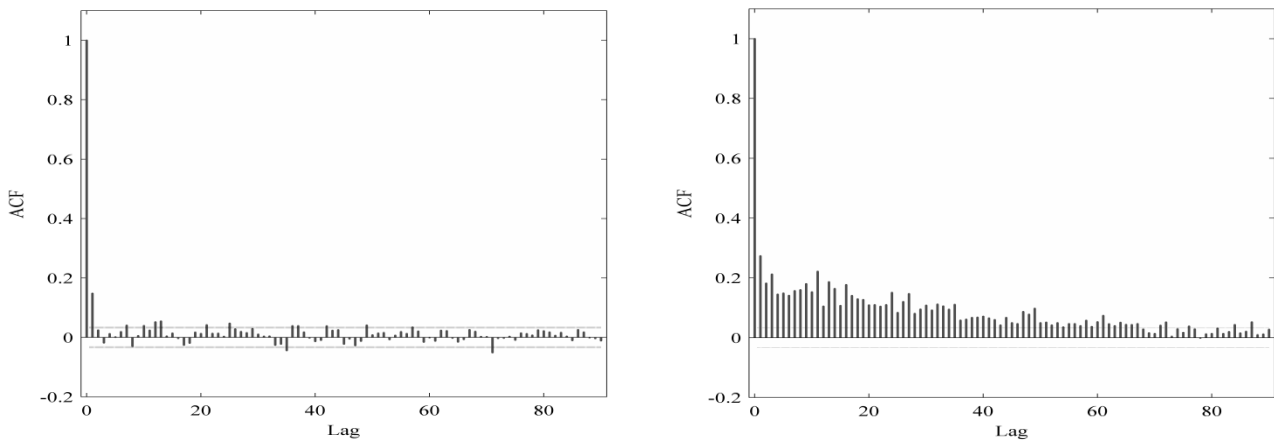


Fig.3: Correlograms for the return data as well as the square return for the RTS

lag	Index	Statistic for $r$	Statistic for $r^2$
30	BUX	81.18	1869.00
	PX	115.32	5499.30
	NSEI	125.53	2017.70
	RTS	153.17	2477.30
	TWII	48.71	1103.80
	SSE	36.04	716.87
	JKSE	159.58	1135.50
60	BUX	142.48	2038.40
	PX	170.50	5658.50
	NSEI	157.36	2216.90
	RTS	209.44	2992.50
	TWII	71.97	1499.50
	SSE	83.66	1199.60
	JKSE	224.60	1385.30
90	BUX	179.94	2160.60
	PX	221.89	5805.60
	NSEI	201.56	2259.10
	RTS	240.91	3120.80
	TWII	107.80	1588.30
	SSE	138.90	1587.70
	JKSE	259.12	1551.00

Table 2: Ljung-Box test statistics for raw data return and square return

For backtesting the models, we use a sliding window and consider 1000 data for prediction of one year forward. We call the first 1000 data “in the sample data” and the proceeding one year data “out of the sample”. In this method we move the window for one year and implement the models again. For each movement new coefficients and parameters are extracted from “in the sample data” and applied to



“out of the sample”. With this approach the dynamic structure of emerging markets is easily captured. Using a simple one estimation of coefficients cannot forecast the future in an appropriate manner, and sliding windows which we recommend in this paper seems to be vigorous enough to satisfy our exceptions.

The VaR estimation under the normal assumption is considerably different from Student’s t-distribution assumption. For implementing the t-distribution in the innovations distribution, we should consider the degree of freedom ( $\nu$ ). For accommodation of excess kurtosis and reflecting relatively high kurtosis, a low value of  $\nu$  and for considering low kurtosis, a relatively high value of  $\nu$  should be chosen.

The determination of threshold is one important step in calculation of GPD parameters (scale and shape parameters). There are several ways like Hill estimator or mean excess function for determination of threshold to use in peaks-over-threshold (POT) approach for extreme events. As McNeil and Frey (2000) recommended, in a long backtest it is not practical to examine the fitted model carefully every day. Hence, the constant value for number of observations violated from the threshold is considered. We always set  $N_u = 100$  which is the 90<sup>th</sup> percentile of the innovation distribution estimated by historical simulation.

For backtesting we compared the forecasted VaR accuracy at 95%, 99% and 99.5% percentiles. Violation for left tail occurs when the realized daily return in the backtesting period is less than the forecasted value of VaR. Table 3 illustrates the number of out of the sample (in front of each index name), theoretically expected number of violations and actual number of violations obtained using different methods. For a comparison of models’ performance, we test whether the violations are significantly different from the expected violations. The p-value for each backtesting as well as Christoffersen backtest (Christoffersen, 1998) for unconditional coverage level is considered.

One part of Christoffersen backtest is the test of the model for generating the “correct frequency” of exceedances, which is called the prediction of correct unconditional coverage. If  $V$  is the number of exceedances in backtest sample, and  $n$  is the total number of observations, then the observed frequency of exceedances is  $V/n$ . Given that the predicted probability of exceedances is  $\alpha$ , the test can be expressed in terms of a likelihood ratio (LR) test. The test statistic  $LR_{UC} = -2\ln\left[\frac{(1-p)^{n-V} p^V}{(1-\frac{V}{n})^{n-V} (\frac{V}{n})^V}\right]$  indicates the correct hypothesis of unconditional coverage if it is distributed as a  $\chi^2(1)$ .

It can be observed from Table 2 that conditional model with EVT passed all our 21 cases, and it had the best performance among others in 12 occasions (joint or alone). The other conditional model with Student’s t-distribution successfully predicted 14 cases, and it had the best accuracy in 2 cases. The semi-parametric approach of FHS had a reasonable performance, and failed in just 3 cases and provided the best estimations in 9 occasions (joint or alone). The results from EVT model are quite satisfactory and confirm the results of previous studies about aptness of EVT model for VaR estimation (McNeil and Frey 2000, Ghorbel and Trabelsi, 2007). On the contrary, t-distribution in parametric approaches because of its weak adaptability to kurtosis of innovation, cannot track the dynamic structure of

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emerging markets so in some cases we do not see the proper performance. Although FHS method has benefits of Historical Simulation and conditional volatility models, it is not as accurate as EVT model, and it has some weaknesses in its adaptation to emerging markets volatilities.

Backtesting Results	BUX(2250)			PX(2750)			RTS(2500)		
	V	p-value	UC	V	p-value	UC	V	p-value	UC

### 0.95 Quantile

Expected	112			137			125		
Conditional t	102	0.33	1.06	157	0.08	2.79	139	0.21	1.60
Conditional EVT	98	0.17	2.05	145	0.49	0.42	115	0.39	0.86
FHS	107	0.64	0.29	150	0.25	1.16	122	0.83	0.08

### 0.99 Quantile

Expected	22			27			25		
Conditional t	26	0.40	0.52	39	0.01	4.30	35	0.04	3.59
Conditional EVT	24	0.69	0.10	32	0.34	0.71	26	0.95	0.04
FHS	25	0.54	0.27	33	0.24	1.04	24	0.95	0.04

### 0.995 Quantile

Expected	11			13			12		
Conditional t	15	0.25	1.14	23	0.00	5.20	15	0.40	0.47
Conditional EVT	14	0.42	0.63	15	0.56	0.11	10	0.59	0.54
FHS	12	0.86	0.05	20	0.03	2.50	11	0.81	0.19

Backtesting Results	JKSE(2000)			TWII(2000)			SSE(1500)			NSEI(3500)		
	V	p-value	UC	V	p-value	UC	V	p-value	UC	V	p-value	UC

### 0.95 Quantile

Expected	100			100			75			175		
Conditional t	111	0.28	1.23	114	0.16	1.98	105	0.00	11.30	190	0.26	1.32
Conditional EVT	95	0.65	0.27	99	0.97	0.01	88	0.13	2.25	186	0.42	0.71
FHS	96	0.73	0.17	107	0.51	0.50	88	0.13	2.25	188	0.33	0.99

### 0.99 Quantile

Expected	20			20			15			35		
Conditional t	31	0.01	5.23	31	0.01	5.23	15	1.00	0.00	42	0.27	1.33
Conditional EVT	17	0.59	0.48	20	1.00	0.00	18	0.53	0.57	41	0.35	0.98
FHS	20	1.00	0.00	28	0.08	2.87	25	0.01	5.61	42	0.27	1.33

### 0.995 Quantile

Expected	10			10			7			17		
Conditional t	14	0.26	1.43	15	0.13	2.18	9	0.48	0.28	31	0.00	8.50
Conditional EVT	10	1.00	0.00	11	0.92	0.10	11	0.12	1.43	21	0.34	0.66
FHS	11	0.92	0.10	12	0.66	0.38	14	0.00	4.50	20	0.48	0.34

Table 3: Expected number of violations and number of violations obtained using different approaches (V), p-values for a binominal test and Unconditional Coverage levels (UC) from Christoffersen test are given in front of each result.

In Fig. 4 part of backtest for RTS index in the period of 9/5/2008 to 9/10/2009 are displayed. This time span is the period that Russia was suffering from consequences of global financial crisis. Our investigation showed that for VaR estimation in 95% percentile in this period, conditional Student's t distribution violated 20 times, and EVT and FHS violated 16 and 27 times respectively. The expected number of violations is about 12 in a year and the high violation of FHS method from expected value is the effect of inadequate flexibility of the model.

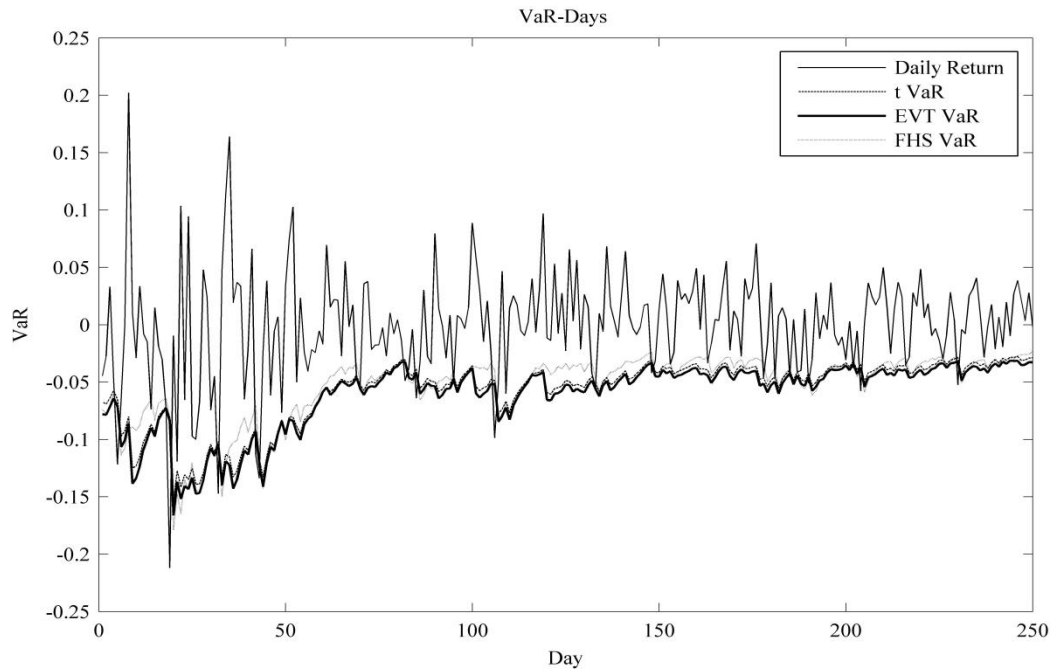


Fig.4: Backtesting of RTS index for estimation of 95% quantile with conditional EVT, conditional t and filtered historical simulation methods.

## 4. Conclusion

In this study, we compared the performance of selected Value at Risk (VaR) methods in forecasting of different percentiles in the left tail of return distribution. We tested VaR models in European and Asian emerging markets. Among variety of parametric and semi-parametric approaches, we fitted a Student's t-distribution on data conditional on a GARCH volatility process to capture both tail heaviness and volatility clustering. In addition, the combination of fitting of GARCH model to estimate the volatility and extreme value theory (EVT) to estimate the tail of the innovation was considered as another parametric approach.

Among our parametric methods, EVT showed relatively the best performance and under no circumstance it failed to pass in the 95% level of confidence. Although GARCH model with t-distribution could take excess kurtosis into account, it failed in 7 out of 21 occasions so it was not considered as a reliable approach. This is different from the results deducted from some previous researches such as McNeil and Frey (2000). The mentioned study considered the developed countries indices such as S&P and DAX which have different characters comparing to the peculiar, dynamic structure of emerging markets such as RTS and SSE.

Another studied method is FHS that is the semi-parametric method with combination of benefits of Historical Simulation and conditional volatility models. This method had medium performance and it failed in 3 cases among our 21 cases of study. It is worth mentioning that FHS was accurate in 95% quantile, but it did not perform precisely in 99% or 99.5%. This finding is somehow different with the result of research by Saša Žiković and Bora Aktan (2009) which deducted that a version of FHS model presents a viable alternative to EVT models. In other words, the characteristic of each market requires different models for accurate VaR estimations, and also as we can deduce from our study as well as other researches, it does not seem that there is a unique model that can perform better than others.

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