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The Joint Settings of Quality Investment, Commission Fee, and Order Quantity under the Consignment Policy

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Chen and Liu (2008) proposed an optimal consignment policy considering a fixed fee and a per-unit commission. However, their model assumed the perfect product and didn't consider the product quality. The objective of this study is to present a modified Chen and Liu's (2008) model by considering quality investment for increasing both manufacturer's and retailer's profits. Taguchi's (1986) quadratic quality loss function is adopted for evaluating the product quality. The optimal parameters of manufacturer's quality investment and commission fee and retailer's economic order quantity are simultaneously determined by maximizing the expected profit of supply chain system. A comparative study between the modified model with/without quality investment is provided for illustration. Numerical results show that the modified Chen and Liu's (2008) model with quality investment has a larger order quantity, a smaller per unit commission, a larger fixed commission fee , a larger manufacturer's expected profit, a larger retailer's expected profit, and a larger expected profit of the supply-chain system than those of one without quality investment.

1. Introduction

In a supply chain system, the trade-off problem between manufacturer and retailer to obtain the maximum expected total profit of society always receive considerable attention. The manufacturer considers the sale, manufacturing, inventory, shipment, and management of products for obtaining the maximum expected profit. Meanwhile, the retailer considers the order quantity, the holding cost, and the goodwill loss for obtaining the maximum expected profit. Recently, Glock (2012) reviews the integrated inventory problems with stochastic demand and stochastic lead time, order/setup cost reduction and lead time reduction, product quality, product deterioration and decay, and learning. Many researchers have addressed the important work for supply chain system with cooperation between the retailer and the manufacturer.

Chen and Liu (2007) presented the optimum profit model between the producers and the purchasers for the supply chain system with pure procurement policy from the regular supplier and with mixed procurement policy from the regular supplier and the spot market. Chen and Liu (2008) further proposed an optimal consignment policy considering a fixed fee and a per-unit commission. Their model determines a higher manufacturer's profit than the traditional production system and coordinates the retailer to obtain a large supply chain profit. However, their model assumed that all of the manufacturer's products are perfect. In fact, it's difficult to provide 100% perfect product in the manufacturer's process. Hence, the product quality loss problem should be considered in their model. Chen and Huang (2010) addressed the problem that the retailers purchase their products from options and online spot markets for hedging the risk of demand uncertainty. For Chen and Liu's (2008) model, they considered the simple manufacturing cost and neglected the

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used cost of customers in traditional production system. Hence, Chen (2010) proposed a modified Chen and Liu’s (2008) model with imperfect product for determining the optimum product and process parameters. Recently, Yu et al. (2012) have presented an extensional work for Chen and Liu’s (2008) model with multi-retailer and generalized demand distributions.

The on-line 100% rectifying inspection can be used as a short-term method for controlling the product quality shipped to the customers. However, quality investment is an available method for improving the process parameters in the long-term. For example, one can buy a new machine for manufacturing the product and address the continuous education training for personnel. Hong et al. (1993), Ganeshan et al. (2001), Chen and Tsou (2003), and Tsou (2006) have presented the declining exponential reduction of process mean and standard deviation as the function of quality investment. Abdul-kader et al. (2010) further adopted Chen and Tsou’s (2003) quality investment function for determining the optimum quality investment and corresponding improved process mean and standard deviation.

Quality investment is a long-term method for improving product quality, decreasing the product’s cost, and promoting the enterprise’s profit. The objective of this study is to present a modified Chen and Liu’s (2008) model by considering quality investment for increasing both manufacturer’s and retailer’s profits. The author proposes a modified Chen and Liu’s (2008) model with imperfect product and quality investment under the consignment policy. The motivation behind this work stems the fact that neglect of product quality loss within the specification limits could overestimate the expected total profit of the system. Long-term quality improvement can promote the service level of product and the customer’s satisfaction. In this study, Taguchi’s (1986) quadratic quality loss function is adopted for evaluating the product quality and represents the real-world quality loss situations. The optimal parameters of manufacturer’s quality investment and commission fee and retailer’s economic order quantity are simultaneously determined by maximizing the expected profit of supply chain system. A comparative study between the modified model with/without quality investment is provided for illustration. In the next section, the notations is given. Then a brief review of modified Chen and Liu’s (2008) model with traditional production system and modified one with imperfect product under the consignment policy are reported. Then the modification of Chen and Liu’s (2008) model is developed by using symmetric quadratic quality loss function and quality investment concept. The numerical example and sensitivity analysis of parameters are subsequently presented for illustration. Some final concluding remarks are then provided based on the analysis.

2. Literature Review

For comparing the difference of profit between the modified Chen and Liu’s (2008) model with imperfect product and quality investment under consignment policy and traditional production system, we firstly need to review the literature about Chen’s (2010) research in this section. Then we can propose the modified model with imperfect product and quality investment in next section.

2.1 Modified Chen and Liu’s Traditional Production System Model

Chen (2010) proposed a modified Chen and Liu’s (2008) model with traditional production system by assuming that the demand of customer (X) and the quality characteristic of product (Y) are independent. The manufacturing cost per unit includes the constant and variable production costs, where the variable production cost per unit is proportional to the value of quality characteristic of product. The 100% inspection of product is executed before shipping to the customer. The non-conformance of product is scrapped and sold at a lower price. Chen (2010) also considered the used cost of product for the retailer by adopting Taguchi's (1986) quadratic quality loss function in his modified model. From Chen (2010), the retailer's expected profit is

$$
\begin{aligned}
&E\left(\pi_r^{PS}\right) \\
&= \{\frac{1}{2\sigma_x}(R+H)\cdot\left[Q^2-\left(\mu_x-\frac{1}{2}\sigma_x\right)^2\right]-\left[Q(W+H)\cdot\frac{1}{\sigma_x}\left(Q-\mu_x+\frac{1}{2}\sigma_x\right)\right] \\
&\quad\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]-\frac{1}{2\sigma_x}\left[Q^2-\left(\mu_x-\frac{1}{2}\sigma_x\right)^2\right] \\
&\quad\cdot\int_L^U Loss(y)f(y)dy+(R-W+S)\cdot Q\cdot\left(\mu_x+\frac{1}{2}\sigma_x-Q\right)\cdot\frac{1}{\sigma_x} \\
&\quad\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]-S\cdot\frac{1}{2\sigma_x}\cdot\left[(\mu_x+\frac{1}{2}\sigma_x)^2-Q^2\right]\cdot \\
&\quad\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]-\frac{1}{\sigma_x}\cdot\left[\mu_x+\frac{1}{2}\sigma_x-Q\right]Q\cdot\int_L^U Loss(y)f(y)dy
\end{aligned} \tag{1}
$$

where

$$
\begin{aligned}
&\int_L^U Loss(y)f(y)dy \\
&= k\{\left[(\mu_y-y_0)^2+\sigma_y^2\right]\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] \\
&\quad+\sigma_y[(\mu_y-2y_0+L)\phi\left(\frac{L-\mu_y}{\sigma_y}\right)-\left(\mu_y-2y_0+U\right)\phi\left(\frac{U-\mu_y}{\sigma_y}\right)]\}
\end{aligned} \tag{2}
$$

The manufacturer's profit may be given by

$$
\pi_m^{PS}=\begin{cases} W-b-cY-i, & L\le Y\le U \\ S_p-b-cY-i, & Y<L \text{ or } Y>U \end{cases} \tag{3}
$$

Then, the manufacturer's expected profit is

$$E\left(\pi_m^{PS}\right) = Q \cdot \{W - \frac{b + c\mu_y + i - S_p\left[1 - \Phi\left(\frac{U-\mu_y}{\sigma_y}\right) + \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]}{\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)}\}$$

(4)

From Chen (2010), the optimum order quantity Q^* for the retailer and the optimum selling price W^* for the manufacturer are as follows:

$$Q^* = \frac{\left[A - (R+H+S)B\right]\mu_x + \frac{1}{2}\left[A - (R+S-H-2W)B\right]\sigma_x}{A-(R+H+S)B}$$

(5)

$$W^* = \frac{\frac{1}{2}\sigma_x\left[(S+R-H+2t)B - A\right] + \mu_x\left[(R+H+S)\cdot B - A\right]}{2B\sigma_x}$$

(6)

where

$$A = k\{\left[(\mu_y - y_0)^2 + \sigma_y^2\right]\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] + \sigma_y[(\mu_y - 2y_0)\phi\left(\frac{L-\mu_y}{\sigma_y}\right) - \left(\mu_y - 2y_0 + U\right)\phi\left(\frac{U-\mu_y}{\sigma_y}\right)]\} \quad (7)$$

$$B = \Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)$$

(8)

$$t = \frac{b + c\mu_y + i - S_p\left[1 - \Phi\left(\frac{U-\mu_y}{\sigma_y}\right) + \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]}{\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)}$$

(9)

The trade-off model between Eqs. (1) and (4) needs to obtain the optimal order quantity (Q^*), the optimal wholesale price (W^*), and the optimal process quality level (μ_y^*) with the maximum expected profits for the retailer and the manufacturer. Substituting Eqs. (5) and (6) into Eq. (4) leads to only one unknown decision variable

μ_y to be determined. Let $L < \mu_y < U$. The direct search method may be applied for obtaining the optimal μ_y^* with the maximum manufacturer's expected profit for the given order quantity and wholesale price. By substituting this optimal μ_y^* into Eqs. (5) and (6), the optimal solution with the maximum expected profit for the retailer and the manufacturer can be obtained.

2.2 Modified Chen and Liu's Consignment Policy with Imperfect Product

To simplify mathematical manipulations, some assumptions are made in the modified Chen and Liu's (2008) model as follows:

1. The demand of customer (*X*) and the quality characteristic of product (*Y*) are independent.
2. The variable production cost per unit is proportional to the value of quality characteristic of product.
3. Imperfect products occur in the production system and the 100% inspection of product
   is executed before shipping to the customer.
4. The non-conformance of product is scrapped and sold at a lower price.
5. The used cost of product is measured by adopting Taguchi's quadratic quality loss function.
6. The manufacturer guarantees that the retailer will be at least as well-off as in the traditional production system.

By applying Taguchi's (1986) quadratic quality loss function, the retailer's profit may be expressed by

$$\pi_r^{CP} = \begin{cases} \alpha X - h_r(Q-X) + A_1 - X \cdot Loss(Y), & \mu_x - \frac{\sigma_x}{2} \le X < Q, L \le Y \le U \\ \alpha Q - S_r(X-Q) + A_1 - Q \cdot Loss(Y), & Q \le X \le \mu_x + \frac{\sigma_x}{2}, L \le Y \le U \end{cases} \tag{10}$$

Then the retailer's expected profit can be obtained as follows:

$$E\left(\pi_r^{CP}\right)$$

$$= \{\frac{1}{2\sigma_x}(\alpha + h_r)\cdot\left[Q^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - \left[Qh_r \frac{1}{\sigma_x}\left(Q - \mu_x + \frac{1}{2}\sigma_x\right)\right]$$

$$- s_r \cdot \frac{1}{2\sigma_x}\cdot\left[(\mu_x + \frac{1}{2}\sigma_x)^2 - Q^2\right] + (\alpha + s_r)\frac{1}{\sigma_x}\cdot\left[\mu_x + \frac{1}{2}\sigma_x - Q\right]Q + A_1\}\cdot B$$

$$- \frac{A}{2\sigma_x}\left[Q^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - Q\cdot\left(\mu_x + \frac{1}{2}\sigma_x - Q\right)\cdot\frac{A}{\sigma_x}$$

(11)

where

$$A = k\{\left[(\mu_y - y_0)^2 + \sigma_y^2\right] \cdot \left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] + \sigma_y[(\mu_y - 2y_0 + L)\phi\left(\frac{L-\mu_y}{\sigma_y}\right)$$

$$-\left(\mu_y - 2y_0 + U\right)\phi\left(\frac{U-\mu_y}{\sigma_y}\right)]\} \qquad (12)$$

$$B = \Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)$$

(13)

Letting the partial derivative of Eq. (11) with respect to *Q* be zero, i.e., $\frac{\partial E(\pi_r^{CP})}{\partial Q} = 0$, results in the optimal order quantity for the retailer as follows:

$$Q_r^* = (\mu_x - \frac{\sigma_x}{2}) + \frac{[(\alpha + s_r)B - A]\sigma_x}{[(\alpha + s_r + h_r)B - A]}$$

(14)

The profit of the supply chain system can be given by

$$\pi_t^{CP} = \begin{cases} RX - (b + cy + i)Q - h_t(Q - X) - X \cdot Loss(Y), & \mu_x - \frac{\sigma_x}{2} < X < Q, L \le Y \le U \\ RQ - (b + cy + i)Q - s_t(X - Q) - Q \cdot Loss(Y), & Q \le X \le \mu_x + \frac{\sigma_x}{2}, L \le Y \le U \\ (S_p - b - cy - i)Q, & \mu_x - \frac{\sigma_x}{2} < X < \mu_x + \frac{\sigma_x}{2}, Y < L \text{ or } Y > U \end{cases} \qquad (15)$$

Therefore, the supply chain system's expected profit can be obtained as follows:

$$E\left(\pi_t^{CP}\right)$$

$$= \{\frac{1}{2\sigma_x}\left(R + h_t\right) \cdot \left[Q^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - \left[Qh_t \frac{1}{\sigma_x}\left(Q - \mu_x + \frac{1}{2}\sigma_x\right)\right]$$

$$- s_t \cdot \frac{1}{2\sigma_x} \cdot \left[(\mu_x + \frac{1}{2}\sigma_x)^2 - Q^2\right] + (R + s_t)\frac{1}{\sigma_x} \cdot \left[\mu_x + \frac{1}{2}\sigma_x - Q\right]Q\} \cdot B$$

$$- \frac{A}{2\sigma_x}\left[Q^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - Q \cdot \left(\mu_x + \frac{1}{2}\sigma_x - Q\right) \cdot \frac{A}{\sigma_x} + S_p(1 - B)Q - (b + c\mu_y + i)Q$$

(16)

Letting the partial derivative of Eq. (16) with respect to *Q* be zero, i.e., $\frac{\partial E(\pi_t^{CP})}{\partial Q} = 0$,

leads to the optimal order quantity for the supplier chain system as follows:

$$Q_t^* = (\mu_x - \frac{\sigma_x}{2}) + \frac{[(R + s_r)B - A + Z_{11}]\sigma_x}{[(R + s_t + h_t)B - A]}$$

(17)

where

$$Z_{11} = S_p(1 - B) - b - c\mu_y - i$$

(18)

Setting $Q_r^* = Q_t^*$ gives the optimal per-unit commission as follows:

$$\alpha^* = -s_r + \frac{[(R + s_t)B - A + Z_{11}]h_r B + A(h_t B - Z_{11})}{(h_t B - Z_{11})B}$$

(19)

The profit of the manufacturer can be expressed by:

$$\pi_m^{CP} = \begin{cases} (R - \alpha)X - (b + cy + i)Q - h_m(Q - X) - A_1, & X < Q, L \le Y \le U \\ (R - \alpha)Q - (b + cy + i)Q - s_m(X - Q) - A_1, & X \ge Q, L \le Y \le U \\ (S_p - b - cy - i)Q, & \mu_x - \frac{\sigma_x}{2} < X < \mu_x + \frac{\sigma_x}{2}, Y < L \text{ or } Y > U \end{cases} \quad (20)$$

Thus, the manufacturer's expected profit can be obtained as follows

$$\begin{aligned} &E\left(\pi_m^{CP}\right) \\ &= \{\frac{1}{2\sigma_x}\left(R - \alpha + h_m\right) \cdot \left[Q^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - \left[Qh_m \frac{1}{\sigma_x}\left(Q - \mu_x + \frac{1}{2}\sigma_x\right)\right] \\ &- s_m \cdot \frac{1}{2\sigma_x} \cdot \left[(\mu_x + \frac{1}{2}\sigma_x)^2 - Q^2\right] + (R - \alpha + s_m)\frac{1}{\sigma_x} \cdot \left[\mu_x + \frac{1}{2}\sigma_x - Q\right]Q - A_1\} \cdot B \\ &+ S_p(1 - B)Q - (b + c\mu_y + i)Q \end{aligned}$$

(21)

There are four decision variables in the modified Chen and Liu's (2008) model with consignment policy and imperfect quality, including order quantity (*Q*), per-unit commission (α), fixed commission (A_1), and process mean of product (μ_y). The mathematical model for obtaining the optimal values of these decision variables is as follows:

$$\text{Max } E\left(\pi_m^{CP}\right)$$

(22)

subject to $E\left(\pi_r^{CP}\right) \geq E(\pi_r^{PS})$ (23)

where

$$E\left(\pi_r^{PS}\right)$$

$$= \{\frac{1}{2\sigma_x}\left(R+H\right)\cdot\left[Q_1^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right] - \left[Q_1\left(W_1+H\right)\cdot\frac{1}{\sigma_x}\left(Q_1 - \mu_x + \frac{1}{2}\sigma_x\right)\right]$$

$$\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] - \frac{1}{2\sigma_x}\left[Q_1^2 - \left(\mu_x - \frac{1}{2}\sigma_x\right)^2\right]\}\cdot A$$

$$+(R-W_1+S)\cdot Q_1\cdot\left(\mu_x + \frac{1}{2}\sigma_x - Q_1\right)\cdot\frac{1}{\sigma_x}$$

$$\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] - S\cdot\frac{1}{2\sigma_x}\cdot\left[(\mu_x + \frac{1}{2}\sigma_x)^2 - Q_1^2\right]\cdot$$

$$\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] - \frac{1}{\sigma_x}\cdot\left[\mu_x + \frac{1}{2}\sigma_x - Q_1\right]Q_1\cdot A \qquad (24)$$

$$A = k\{\left[(\mu_y - y_0)^2 + \sigma_y^2\right]\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]$$

$$+\sigma_y[(\mu_y - 2y_0 + L)\phi\left(\frac{L-\mu_y}{\sigma_y}\right) - \left(\mu_y - 2y_0 + U\right)\phi\left(\frac{U-\mu_y}{\sigma_y}\right)]\} \qquad (25)$$

$$Q_1 = \frac{\left[A-\left(R+H+S\right)B\right]\mu_x + \frac{1}{2}\left[A-\left(R+S-H-2W_1\right)B\right]\sigma_x}{A-(R+H+S)B}$$

(26)

$$W_1 = \frac{\frac{1}{2}\sigma_x\left[(S+R-H+2t)B-A\right] + \mu_x\left[\left(R+H+S\right)\cdot B - A\right]}{2B\sigma_x}$$

(27)

$$B = \Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)$$

(28)

$$t=\frac{b+c\mu_y+i-S_p\left[1-\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)+\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]}{\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)}$$

(29)

$$E_1\left(\pi_r^{CP}\right)$$
$$=\{\frac{1}{2\sigma_x}(\alpha+h_r)\cdot\left[Q_t^2-\left(\mu_x-\frac{1}{2}\sigma_x\right)^2\right]-\left[Q_t h_r\frac{1}{\sigma_x}\left(Q_t-\mu_x+\frac{1}{2}\sigma_x\right)\right]$$
$$-s_r\cdot\frac{1}{2\sigma_x}\cdot\left[(\mu_x+\frac{1}{2}\sigma_x)^2-Q_t^2\right]+(\alpha+s_r)\frac{1}{\sigma_x}\cdot\left[\mu_x+\frac{1}{2}\sigma_x-Q_t\right]Q_t\}\cdot B$$
$$-\frac{A}{2\sigma_x}\left[Q_t^2-\left(\mu_x-\frac{1}{2}\sigma_x\right)^2\right]-Q_t\cdot\left(\mu_x+\frac{1}{2}\sigma_x-Q_t\right)\cdot\frac{A}{\sigma_x}$$

(30)

$$Q_t=(\mu_x-\frac{\sigma_x}{2})+\frac{[(R+s_r)B-A+Z_{11}]\sigma_x}{[(R+s_t+h_t)B-A]}$$

(31)

$$Z_{11}=S_p(1-B)-b-c\mu_y-i$$

(32)

$$\alpha=-s_r+\frac{[(R+s_t)B-A+Z_{11}]h_r B+A(h_t B-Z_{11})}{(h_t B-Z_{11})B}$$

(33)

$$E(\pi_r^{CP})=E_1\left(\pi_r^{CP}\right)+A_1 B$$

(34)

$$A_1=[E(\pi_r^{PS})-E_1\left(\pi_r^{CP}\right)]/B$$

(35)

The following solution procedure may be applied for obtaining the optimal solution of Eqs. (22) and (23):

Step 1. Substituting Eqs. (31), (33), and (35) into Eq. (22) results in the only one unknown decision variable, μ_y, in the model.

Step 2. Let $L<\mu_y<U$. One may use the direct search method for obtaining the optimal μ_y^* with the maximum manufacturer's expected profit for the manufacturer.

Step 3. Substitute this optimal μ_y^* from Step2 into Eqs. (31), (33), and (35). Then we can obtain the optimal values of order quantity (Q^*), per-unit commission (α^*), and fixed commission (A_1^*).

Step 4. Substituting the combination of decision variables (Q^*, α^*, A_1^*, μ_y^*) into Eqs. (11), (24), and (16) leads to the optimal expected profit of retailer for consignment policy, the optimal expected profit of retailer for traditional production system, and the optimal expected profit of supply chain system for consignment policy, respectively.

3. Methodology---The Modified Chen and Liu's Consignment Policy with Imperfect Product and Quality Investment

Consider the quality investment can improve the known process mean and standard deviation of product quality characteristic and decrease the bias and variability of product. For example, we can address the quality train for person, use the new material and manufacturing equipment for process. It can improve product output quality, increase the manufacturer's profit, and promote total profit of supply chain. We adopt Chen and Tsou's (2003) quality investment function which has the declining exponential reduction of process mean and standard deviation as the function of quality investment. Hence, the optimum profit model is as follows:

$$\text{Max } E\left(\pi_m^{CP}\right) \tag{36}$$

$$\text{subject to } E\left(\pi_r^{CP}\right) \geq E(\pi_r^{PS}) \tag{37}$$

where

$$\mu_y^2 = \mu_T^2 + (\mu_0^2 - \mu_T^2)\exp(-\beta I) \tag{38}$$

$$\sigma_y^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha I) \tag{39}$$

The constraint of the above mathematical model, Eq. (37), is based on the larger retailer's profit under consignment policy than that of traditional production system. It assures the retailer's benefit when the retailer agrees to adopt the consignment policy. The objective function of this model, Eq. (36), is to obtain the maximum manufacturer's profit by process improvement through quality investment. It is anticipated that a good long-term quality policy will promote the customer's satisfaction on product/service and manufacturer's profit. The trade-off model needs to obtain the optimal order quantity (Q^*) ,per-unit commission (α^*) , fixed commission (A_1^*), and the optimal quality investment (I^*) with the maximum expected profit for the manufacturer under the constraint Eq. (37) satisfied. Substituting Eqs. (24)-(35) and (38)-(39) into Eq. (36), there are one unknown decision variable, I . One can adopt the direct search method for obtaining the optimal I^* with the corresponding improved process mean (μ_y^*) and standard deviation (σ_y^*) under the maximum manufacturer's expected profit. Substitute these optimal μ_y^*, σ_y^*, and I^* values into Eqs. (31), (33), and (35). Then we can obtain the optimal values of order quantity (Q^*), per-unit commission (α^*), and fixed commission (A_1^*). Substituting the combination of decision variables (Q^*, α^*, A_1^*, I^*) into Eqs. (11), (24), and (16) leads to the optimal expected profit of retailer for consignment policy, the optimal expected profit of retailer for traditional production system, and the optimal expected profit of supply chain system with consignment policy, imperfect quality, and quality investment, respectively.

4. Result---Numerical Example and Sensitivity Analysis

The market demand is assumed to have a uniform distribution with mean 100 and variability 200. The numerical example employs financial parameters of R = 30, S = 0.1, H = 0.36, S_p = 0.1, s_r = 0.1, s_m = 0.1, s_t = 0.2, h_t = 0.36, h_m = 0.18, h_r = 0.18, b = 0.1, i = 0.02, c = 1 and quality parameters $\sigma_0 = 0.5, \sigma_T = 0, \mu_0 = 4.5, \mu_T = 5,$ $\alpha = 0.01, \beta = 0.05,$ y_0 = 5, k = 4, L = 3, U = 7. By applying the aforementioned solution procedure for solving Eqs. (36) and (37), we can obtain the optimal process mean $I^* = 108.01$, which results in the optimal order quantity $Q^* = 163$, the per unit commission $\alpha^* = 2.027$ and the fixed commission fee $A_1^* = 416.24$ with the manufacturer's expected profit $E(\pi_m^{CP}) = 1329.43$, the retailer's expected profit for traditional system $E(\pi_r^{PS}) = 469.27,$ the retailer's expected profit for consignment policy $E(\pi_r^{CP}) = 469.27,$ and the expected profit of the supply chain system $E(\pi_t^{CP}) = 1798.71$. If there is no quality improvement for production process, the optimal solution of modified Chen and Liu's (2008) model is: the order quantity $Q^* = 161$, the per unit commission $\alpha^* = 4.123$ and the fixed commission fee $A_1^* = 374.99$ with the manufacturer's expected profit $E(\pi_m^{CP}) = 1206.17,$ the retailer's expected profit for traditional system $E(\pi_r^{PS}) = 392.91,$ the retailer's expected profit for consignment policy $E(\pi_r^{CP}) = 392.91,$ and the expected profit of the supply chain system $E(\pi_t^{CP}) = 1599.07$. In this numerical example, it shows that the modified model with quality investment has larger manufacturer's expected profit, retailer's expected profit, and expected profit of the supply chain system than those of modified one without quality investment.

Tables 1-2 list $\pm 20\%$ magnitude change for parameter values and show their effects on the order quantity, process mean, per unit commission, fixed commission fee, manufacturer's expected profit, retailer's expected profit, and expected profit of the supply chain system for modified Chen and Liu's (2008) model with/without quality investment. A parameter is considered as a major effect on the expected profit if the change magnitudes of manufacturer's, retailer's and supply chain system's expected profits are greater than 10%. From Tables 1-2, we may have the following observations:

1. As the selling price per unit, R, increases, the order quantity, the per unit commission, and the fixed commission fee increase. The selling price per unit has a

major effect on the manufacturer's expected profit, retailer's expected profit, and expected profit of the supply-chain system for the modified Chen and Liu's (2008) model with/without quality investment. By the quality improvement, the manufacturer can sell the product at a higher price and all of the membership in supply chain system obtain a larger expected profit.

Table 1: The Effect of Parameters of Optimal Solution for Modified Chen and Liu's (2008) Model with Quality Investment

R	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
24	153	104.06	1.880	290.70	899.655	324.77	1224.41
30	163	108.01	2.027	416.24	1329.43	469.27	1798.71
36	169	110.38	2.195	543.35	1766.09	615.57	2381.66
μ_x	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
80	142	87.20	2.307	287.17	999.14	325.48	1324.62
100	163	108.01	2.027	416.24	1329.43	469.27	1798.71
120	183	126.70	1.811	502.50	1707.48	570.18	2277.65
σ_x	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
160	150	109.99	2.003	408.78	1423.50	464.86	1888.35
200	163	108.01	2.027	416.24	1329.43	469.27	1798.71
240	175	108.00	2.027	378.72	1280.85	428.85	1709.70
k	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
3.2	163	90.27	1.955	417.73	1350.65	470.44	1821.09
4.0	163	108.01	2.027	416.24	1329.43	469.27	1798.71
4.8	163	12.69	2.054	415.68	1311.45	468.83	1780.28
b	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	163	107.99	2.031	416.71	1331.87	470.08	1801.95
0.10	163	108.01	2.027	416.24	1329.43	469.27	1798.71
0.12	163	107.90	2.025	415.74	1327.00	468.42	1795.42
c	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.8	170	109.39	2.225	437.26	1453.84	511.16	1965.00
1.0	163	108.01	2.027	416.24	1329.43	469.27	1798.71
1.2	156	106.37	1.900	390.24	1210.17	429.02	1639.19
i	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.016	163	108.01	2.028	416.34	1329.92	469.43	1799.35
.020	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.024	163	108.03	2.026	416.15	1328.94	469.12	1798.06
S_p	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	163	108.03	2.027	416.25	1329.43	469.28	1798.71
0.10	163	108.01	2.027	416.24	1329.43	469.27	1798.71
0.12	163	108.03	2.027	416.25	1329.43	469.28	1798.71
s_r	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	163	108.06	2.047	414.25	1329.50	469.29	1798.79
0.10	163	108.01	2.027	416.24	1329.43	469.27	1798.71
0.12	163	108.05	2.007	418.25	1329.36	469.29	1798.65
s_m	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	163	108.01	2.027	416.24	1329.50	469.27	1798.78
0.10	163	108.01	2.027	416.24	1329.43	469.27	1798.71
0.12	163	108.21	2.025	416.30	1329.36	469.34	1798.70
s_t	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.16	163	108.17	2.024	416.41	1329.43	469.33	1798.76
.20	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.24	163	108.17	2.027	416.16	1329.43	469.33	1798.76

Table 1 (Continued)

h_r	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.144	163	108.06	1.871	428.92	1331.81	469.29	1801.10
.180	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.216	163	107.83	2.185	403.53	1327.06	469.21	1796.27
h_m	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.144	163	108.21	2.025	416.30	1331.81	469.34	1801.15
.180	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.216	163	108.06	2.027	416.25	1327.06	469.29	1796.35
h_t	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.288	163	107.87	2.039	415.20	1329.42	469.32	1798.65
.360	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.432	162	108.01	2.017	417.22	1329.42	469.27	1798.69
S	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	163	108.01	2.027	417.43	1327.92	470.79	1798.71
0.10	163	108.01	2.027	416.24	1329.43	469.27	1798.71
0.12	163	108.01	2.027	415.06	1330.95	467.76	1798.71
H	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.288	163	108.01	2.027	417.43	1328.24	470.46	1798.71
.360	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.432	163	108.01	2.027	415.06	1330.62	468.09	1798.71
α_1	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.008	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.010	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.012	163	108.01	2.027	416.24	1329.43	469.27	1798.71
β_1	Q	I	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.04	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.05	163	108.01	2.027	416.24	1329.43	469.27	1798.71
.06	163	108.01	2.027	416.24	1329.43	469.27	1798.71

2. As the mean of the demand of customer, μ_x, increases, the order quantity increases, the per unit commission decreases, and the fixed commission fee increases. The mean of the demand of customer is a major effect to the manufacturer's expected profit, retailer's expected profit, and expected profit of the supply-chain system for the modified Chen and Liu's (2008) model with/without quality investment. Hence, the increase of mean demand of customer can promote a larger expected profit for all of the membership in supply chain system

3. The mean of the demand of customer, μ_x, and the quality loss coefficient, *k*, have the major effects on the quality investment, *I*. Hence, the mean demand of customer and the cost of quality improvement are two important factors for consideration in deciding the long-term quality investment policy.

4. The modified Chen and Liu's (2008) model with quality investment has a larger order quantity, a smaller per unit commission, a larger fixed commission fee increase, a larger manufacturer's expected profit, a larger retailer's expected profit,

and a larger expected profit of the supply-chain system than those of one without quality investment. Hence, the quality investment policy should be a correct method for maintaining the good long-term relationship between the manufacturer and the retailer

Table 2: The Effect of Parameters of Optimal Solution for Modified Chen and Liu's (2008) Model without Quality Investment

R	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
24	150	3.920	251.77	809.798	260.86	1070.65
30	161	4.123	374.99	1206.17	392.91	1599.07
36	168	4.325	500.59	1610.07	527.50	2137.57
μ_x	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
80	141	4.123	261.25	924.09	275.74	1199.83
100	161	4.123	374.99	1206.17	392.91	1599.07
120	181	4.123	448.27	1525.74	472.58	1998.31
σ_x	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
160	149	4.123	368.22	1289.19	389.31	1678.50
200	161	4.123	374.99	1206.17	392.91	1599.07
240	173	4.123	339.62	1162.20	357.44	1519.64
k	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
3.2	162	3.448	389.51	1252.89	408.46	1661.31
4.0	161	4.123	374.99	1206.17	392.91	1599.07
4.8	160	4.797	360.51	1159.53	377.38	1536.96
b	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	161	4.126	375.50	1208.58	393.71	1602.28
0.10	161	4.123	374.99	1206.17	392.91	1599.07
0.12	161	4.119	374.48	1203.76	392.10	1595.86
c	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.8	168	4.326	395.14	1317.01	429.81	1746.82
1.0	161	4.123	374.99	1206.17	392.91	1599.07
1.2	154	3.982	350.34	1100.16	357.62	1457.78
i	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.016	161	4.123	375.09	1206.65	393.07	1599.71
.020	161	4.123	374.99	1206.17	392.91	1599.07
.024	161	4.122	374.89	1205.68	392.75	1598.43
S_p	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	161	4.122	374.95	1205.99	392.85	1598.84
0.10	161	4.123	374.99	1206.17	392.91	1599.07
0.12	161	4.123	375.03	1206.34	392.96	1599.31
s_r	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	161	4.143	372.99	1206.24	392.91	1599.14
0.10	161	4.123	374.99	1206.17	392.91	1599.07
0.12	161	4.103	376.99	1206.09	392.91	1599.00
s_m	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	161	4.123	374.99	1206.24	392.91	1599.14
0.10	161	4.123	374.99	1206.17	392.91	1599.07
0.12	161	4.123	374.99	1206.09	392.91	1599.00

Table 2 (Continued)

s_t	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.16	161	4.121	375.12	1206.17	392.91	1599.07
.20	161	4.123	374.99	1206.17	392.91	1599.07
.24	161	4.124	374.86	1206.17	392.91	1599.07
h_r	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.144	161	3.976	386.76	1208.32	392.91	1601.22
.180	161	4.123	374.99	1206.17	392.91	1599.07
.216	161	4.269	363.22	1204.01	392.91	1596.92
h_m	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.144	161	4.123	374.99	1208.32	392.91	1601.22
.180	161	4.123	374.99	1206.17	392.91	1599.07
.216	161	4.123	374.99	1204.01	392.91	1596.92
h_t	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.288	161	4.133	374.03	1206.15	392.91	1599.06
.360	161	4.123	374.99	1206.17	392.91	1599.07
.432	161	4.113	375.93	1206.15	392.91	1599.06
S	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
0.08	161	4.123	376.51	1204.76	394.31	1599.07
0.10	161	4.123	374.99	1206.17	392.91	1599.07
0.12	161	4.123	373.47	1207.57	391.50	1599.07
H	Q	α	A_1	$E(\pi_m^{CP})$	$E(\pi_r^{CP})$	$E(\pi_t^{CP})$
.288	161	4.123	376.15	1205.09	393.98	1599.07
.360	161	4.123	374.99	1206.17	392.91	1599.07
.432	161	4.123	373.84	1207.24	391.84	1599.07

5. Conclusions

The sensitivity analysis shows that the selling price per unit and the mean of the demand of customer have a major effect on the manufacturer's expected profit, retailer's expected profit, and expected profit of the supply-chain system. Hence, the exact estimation on these parameters always requires to avoid any incorrect outcome. The quality investment can promote the expected profit for all of the membership in the supply chain system. The managerial implication of this paper is that the joint optimization of quality investment, commission fee, and order quantity management can improve the product/service quality shipped to the customer and increase the expected profit of the supply chain system. Hence, the appropriate quality investment policy can maintain the strong relationship for the manufacturer, retailer, and customer. It can bring more benefit for them.

In the present paper, we present a modified Chen and Liu's (2008) model with consignment policy, imperfect quality, and quality investment, which may be considered as a generation of the original Chen and Liu's (2008) model. The order quantity, per unit commission, fixed commission, and quality investment are simultaneously determined in the modified model. The extension to the modified Chen and Liu's (2008) model with multi-retailers and multi-attribute quality characteristics may be left for further study.

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Appendix

Notations

α the per-unit commission paid by the manufacturer to the retailer
Q the order quantity by the retailer
h_r the per-unit carrying cost for the retailer
s_r the goodwill loss per unit of stock-out for the retailer
X the stochastic demand of customer having an uniform distribution, $X \sim U\left[\mu_x - \frac{1}{2}\sigma_x, \mu_x + \frac{1}{2}\sigma_x\right]$
μ_x the mean demand of customer
σ_x the variability of customer demand
A_1 the fixed commission fee paid by the manufacturer to the retailer
R the per-unit retailer price
c the per-unit manufacturing cost/ the variable production cost per unit
h_t the per-unit carrying cost in the supply chain system
s_t the goodwill loss per unit of stock-out in the supply chain system
h_m the per-unit carrying cost for the manufacturer
s_m the goodwill loss per unit of stock-out for the manufacturer
H the carrying cost per unit for the retailer in the traditional production system
S the goodwill loss per unit of stock-out for the retailer in the traditional production system
μ_0 the known process mean of quality characteristic of product
σ_0 the known process standard deviation of quality characteristic of product
$\mu_T (= y_0)$ the target value of process mean for product quality characteristic
σ_T the target value of process standard deviation for product quality characteristic
Y the quality characteristic of product, which is assumed that $Y \sim N(\mu_y, \sigma_y^2)$
μ_y the improved mean of quality characteristic of product
σ_y the improved standard deviation of quality characteristic of product
I the quality improvement
β_1 the parameter of declining exponential function for the improved process standard deviation
α_1 the parameter of declining exponential function for the improved process mean.
L the lower specification limit of product quality characteristic
U the upper specification limit of product quality characteristic
$Loss(Y)$ the quality loss per unit, $Loss(Y) = k(Y - y_0)^2$
k the quality loss coefficient
$\Phi(\cdot)$ the cumulative distribution function of standard normal random variable
$\phi(\cdot)$ the probability density function of standard normal random variable
W the selling price per unit for the conformance product in the traditional production system
S_p the discounted price per unit for the non-conformance product scrapped in the traditional production system

b the constant production cost per unit
i the inspection cost per unit
π_m^{CP} the manufacturer's profit for consignment policy
$E\left(\pi_m^{CP}\right)$ the manufacturer's expected profit for consignment policy
π_r^{CP} the retailer's profit for consignment policy
$E\left(\pi_r^{CP}\right)$ the retailer's expected profit for consignment policy
π_t^{CP} the supply chain system's profit for consignment policy
$E\left(\pi_t^{CP}\right)$ the supply chain system's expected profit for consignment policy
π_m^{PS} the manufacturer's profit in the traditional production system
$E\left(\pi_m^{PS}\right)$ the manufacturer's expected profit in the traditional production system
π_r^{PS} the retailer's profit in the traditional production system
$E\left(\pi_r^{PS}\right)$ the retailer's expected profit in the traditional production system.